A Non-perturbative Effect in Deep Inelastic Scattering

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Abstract

The HERA data at large Q^2 and small-x investigate large distances on the light-cone. At such large distances the scattered quarks can maintain their colour identity by polarizing the vacuum as they transfer energy to it. We calculate the probability for the creation of quark-antiquark pairs from the polarized QCD vacuum and their contribution to the structure function.

The new experiments at HERA [1, 2] extend the kinematic regions of deep inelastic scattering to much higher values of Q^2 and smaller values of the Bjorken scattering variable x. We review in this article the space-time structure of the scattering and show that it involves kinematic regions where confinement effects are important. This motivated me to consider a new contribution to the structure functions at small x. The analysis considers the rise of $F_2(x)$ at small x and investigates the additional quanta which are excited.

Deep inelastic scattering studies the tensor

$$W_{\mu\nu}(q\cdot p, q^2) = \frac{1}{2\pi}\int d^4y\, e^{iq\cdot y}\langle p|[J_\mu(y),\ J_\nu(0)]|p\rangle\,. \tag{1}$$

The phase of the Fourier transform becomes stationary when

$$y_- = y_0 - y_3 \sim \pm\frac{1}{q_0+q_3} \quad \text{and} \quad y_+ = y_0 + y_3 \sim \pm\frac{1}{q_0-q_3} \tag{2}$$

which, for the time-like distances investigated by the currents, imply

$$y^2 = y_+y_- \ - y_1^2 - y_2^2 \leq y_0^2 - y_3^2 \sim \frac{1}{Q^2}\,. \tag{3}$$

Thus for large Q^2, which is the case at HERA, y^2 is very close to the light cone. This allows the replacement of the commutator by its light-cone singularity times a bilocal operator, i.e.,

$$W_{\mu\nu}(q\cdot p,\ q^2) = s_{\mu\nu\alpha\beta}\int d^4y\, e^{iq\cdot y}\frac{\partial}{\partial y^\alpha}\,\Delta_F(y)\,\langle p|\bar{q}(y)\gamma^\beta q(0)|p\rangle\,. \tag{4}$$

Now following standard techniques, it can be shown that Bjorken's scaling follows [5]. This prediction was studied extensively. Significant violations of scaling have been observed and explained as perturbative corrections from QCD. The new data also indicate that the structure function $F_2(x, Q^2)$ increases by a factor of almost two or more as x decreases, signaling the creation of additional quanta. An open issue is still the description of the quanta created at small x. To this end we note that the distance y_3 along the light-cone becomes very large [3, 4]:

$$y_3 = \pm\frac{1}{2}\,\frac{1}{q_0-q_3} \approx \pm\frac{1}{2M_px} = 2L(x)\,. \tag{5}$$

The distance $L(x)$ depends on the scaling variable and for small x becomes very long in comparison to the size of the proton. Viewing eq. (1) as Compton scattering of two

currents on a proton, we are forced to accept that the distance between the two currents is many times the radius of the proton. Consequently, as the current deposits energy and momentum on a quark q', it forces it to accelerate and travel a long distance ($\sim \frac{1}{4\,M_p \cdot x} \gg \frac{1}{M_p}$). In the course of this journey, q' transfers part of its energy to the vacuum, e.g. by emission of a large number of soft gluons, thus polarizing it. Consequently, a chromoelectric flux tube of length $\sim \frac{1}{4\,M_p x}$ is created between the scattered quark q' and the rest of the (constituent) spectator quarks, forcing the quark q' to remain confined. This tube eventually breaks down into two disjoint tubes when a $q\bar{q}$ pair is created in the field of the tube by the Schwinger mechanism. Creation of further pairs creates additional breaks, eventually resulting in hadronization. The number of hadrons is, roughly, proportional to the number of created $q\bar{q}$ pairs (multiplicity). In this article we study the creation of additional quark-antiquark pairs from a constant chromoelectric and chromomagnetic field [6, 7, 8].

We represent the states as the product of the scattered quark plus a number of quark-antiquark pairs in the presence of the QCD vacuum $|\Omega\rangle$. The final state, before hadronization, can be written schematically as

$$\langle X' q'| = \langle \Omega| \{ < q'| \, + \, \langle q\bar{q}| < q'| \, + \ldots \, \langle n(q\bar{q})| < q'| \} \tag{6}$$

where $< q'|$ is the scattered quark, $\langle\Omega|$ the non-perturbative QCD-vacuum and $\langle n(q\bar{q})|$ a state of n-pairs created by the Schwinger mechanism. Similarly the initial state is presented as $|q, \Omega\rangle$. We show the scattering process in fig. 1, where the pairs are created from the vacuum. Energy is transferred to the vacuum through the emission of many soft gluons, whose detailed description is not yet available. Later on we shall substitute the vacuum by a constant chromoelectric and chromomagnetic field.

I describe next the various terms and processes generated by the S-matrix of the problem. The general S-matrix is

$$S = \mathrm{T}\, exp \left\{ -ie \int A_\mu(y)\, j^\mu(y)\, d^4y - ig \int G^\alpha_\nu(y)\, j^{\nu,\alpha}(y)\, d^4y \right\} \tag{7}$$

with $A_\mu(y)$ and $G^\alpha_\nu(y)$ the electromagnetic and gluonic fields coupled to the corresponding currents $j^\mu(y) = \bar{q}(y)\gamma^\mu q(y)$ and $j^{\nu,\alpha}(y) = \bar{q}(y)\frac{\lambda^\alpha}{2}\gamma^\nu q(x)$, respectively. The capital T indicates the time-ordered product. We shall treat the electromagnetic interaction perturbatively and expand its exponential in a series keeping only the term linear in $A_\mu(y)$.

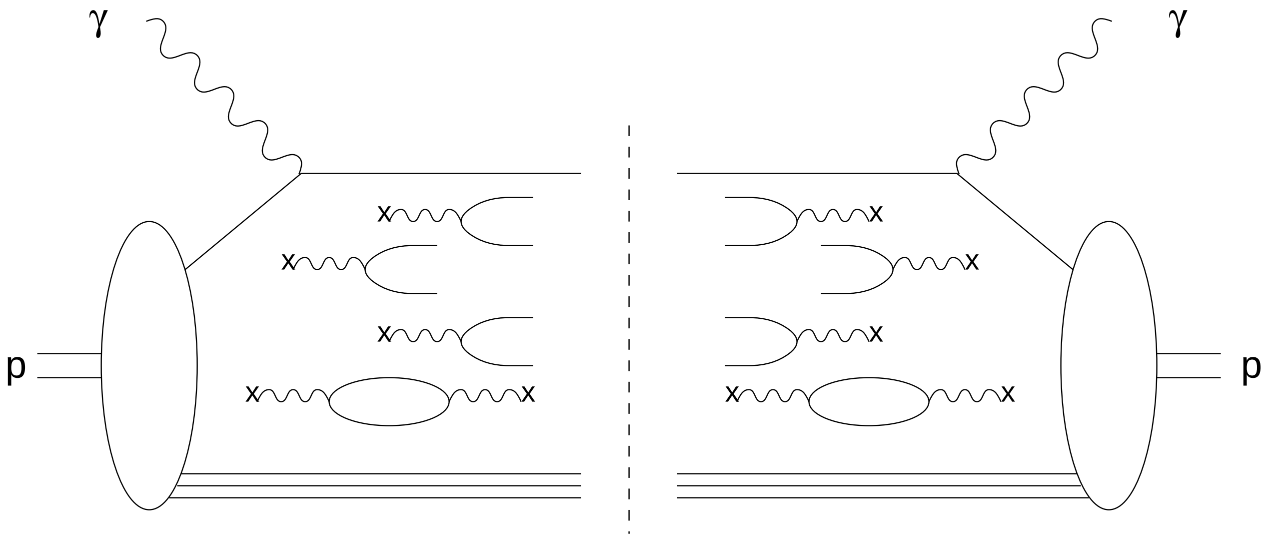

Figure 1: The non-perturbative QCD vacuum with the creation of quark-antiquark pairs.

This can be proven to hold for the time-ordered product. We write the matrix element as

$$\langle X', q'|S|q\rangle = \langle X'q'|\,\mathrm{T}\,(-ie)\int A_\mu(y)j^\mu(y)d^4y\ exp\left\{-ig\int G^\alpha_\nu(z)j^{\nu,\alpha}(z)d^4z\right\}|q,\Omega\rangle. \quad (8)$$

Contractions between the quark fields in $j^\mu(x)$ and $j^{\mu,\alpha}(y)$ produce QCD corrections which are frequently included by summing up the leading logarithmic terms [9]. The perturbative corrections are characterized by the emission of a few hard gluons from the scattered quark q', either before or after its collision with the photon. Therefore, the quark q', after the emission of these hard gluons and the collision with the photon, does not create a long QCD tube. This part of the quark distribution function, whose Q^2-dependence is determined by the hard-gluon perturbative effects, will be represented in eq. (17) by $q_i^{pert}(x, Q^2)$. The distribution by itself is a non-perturbative quantity; only its Q^2-dependence is determined by perturbative QCD.

The new contribution is produced by another class of diagrams generated also through eq. (8), namely those terms which have no contractions between the quark fields in $j^\mu(y)$ and the quark fields in $j^{\nu,\alpha}(y)$. They involve the contractions of quarks in $j^\mu(y)$ with quarks in the initial and final states. This permits us to approximate the new matrix

element by

$$\langle X', q'|S|q\rangle_{n-p} \approx \langle q'| - ie \int A_\mu(y) j^\mu(y) d^4y |q\rangle \; \langle X'|\mathrm{T}\, exp\left\{-ig \int G^\alpha_\nu(z) j^{\nu,\alpha}(z) d^4z\right\} |\Omega\rangle \tag{9}$$

with the subscript $n-p$ indicating that we treat the strong interaction term non--perturbatively. As mentioned already, the quarks in the current $j^\mu(y)$ emit many soft gluons which create the background field (flux-tube). This field then is used as the classical field in the QCD vacuum. The new multiplicative factor from QCD involves the transition of the non-perturbative vacuum $|\Omega\rangle$ to any number of final quark pairs. This transition probability is related to the amplitude of emitting no pairs, i.e., the vacuum persistence probability

$$S_0 = \langle\Omega| exp\left\{-ig \int G^\alpha_\nu(y) j^{\nu,\alpha}(y) d^4y\right\} |\Omega\rangle \,.$$

We can write

$$|S_0|^2 = exp\left\{-\int d^4y \; \omega(y)\right\} \tag{10}$$

and identify $\omega(y)$ with the probability for creating a pair per unit volume of the flux-tube and per unit time. The exact solution of this problem for a constant electric field was obtained by Schwinger [6, 7]. We adopt this problem for our flux-tube and consider the potential

$$G^\alpha_\mu(y) = (0,\, y_3 gB,\, 0,\, tgE)\, \eta^\alpha \tag{11}$$

to represent the background field with η^α a unit vector in color space. It corresponds to a chromoelectric field gE along the direction of the tube and a chromomagnetic field gB perpendicular to the tube. We found the solution [10]

$$\omega(y, m^2) = \alpha_s \frac{|E|\,|B|}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \coth\left(n\frac{|B|}{|E|}\pi\right)\, e^{-\frac{nm^2\pi}{gE}} \tag{12}$$

which in the limit $n\frac{|B|}{|E|}\pi \ll 1$ reduces to the Schwinger solution [6, 7]. The non-perturbative nature of the solution is manifested in the exponential function, which has an essential singularity at $gE \to 0$.

The picture which emerges so far is a flux-tube of unspecified transverse dimensions, but whose length is related to the scaling variable through eq. (5). The quark-antiquark pairs are created in the tube with the probability density given by eq. (12). The sum

of all the pairs modifies the quark distribution functions by a multiplicative factor. This will be a new contribution to be added to the perturbative effects, because, as explained already, the two terms originate from different contractions in eq. (8).

We compute next the probability for creating all possible pairs. To this end we partition the flux tube into small volume elements, as shown in fig. 2. The probability of producing a pair in the element dy_i and no pair in the rest of the tube is

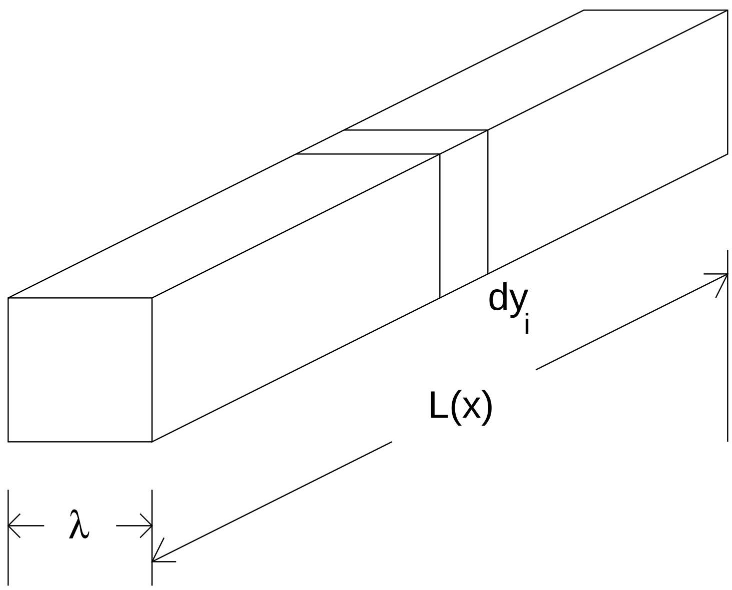

Figure 2: A schematic drawing of the tube, where pairs and fluctuations are created by the background field before and after the interaction with the proton.

$$dP_1 = \lambda^2 T\omega(y_i)\,dy_i \prod_{K=1}^{\infty}\left(1 - \lambda^2 T\omega(y_K)\,dy_K\right)\ . \tag{13}$$

We denote the transverse dimension of the tube by λ and the time for the creation of a pair by T. These are two new parameters to be specified later on. The probability for producing one pair anywhere in the tube is

$$P_1 = \lambda^2 T \int_0^{L(x)} \omega(y)dy\ e^{-\lambda^2 T \int \omega(y)dy}\ . \tag{14}$$

We can generalize this result for n-pairs in the tube

$$P_n(x) = \frac{1}{n!}\left[\lambda^2 T \int_0^{L(x)} \omega(y)dy\right]^n e^{-\lambda^2 T \int_0^{L(x)} \omega(y)dy} \,. \tag{15}$$

Finally, the sum over all possible pairs gives

$$\sum_{n=1}^{\infty} P_n(x) = 1 - e^{-\lambda^2 T \int_0^{L(x)} \omega(y)dy} \,. \tag{16}$$

This correction must be multiplied by the distribution function as it follows from eqs. (8) and (9). Thus each quark distribution function has two components: a perturbative term, which is frequently used to analyze the data [9], plus a new non-perturbative term from the creation of pairs :

$$q_i^{total}(x) = q_i^{pert.}(x, Q^2) + q_i^0(x_p, Q^2)\left[1 - e^{-\lambda^2 T \int_0^{L(x)} \omega(y)dy}\right] \tag{17}$$

The second quark distribution function $q_i^0(x_p, Q^2)$ is generated by the electromagnetic term in eq. (9), where the QCD matrix element is a multiplicative factor. This distribution function is not modified by QCD corrections, a property indicated by the superscript 0. For this reason we expect it to be practically independent of Q^2. Its numerical value is determined at an intermediate value of $x = x_p$ where the flux-tube begins to form. Since there are no gluons radiated by the quark we also expect $q_i^0(x_p, Q^2)$ to remain constant as x decreases. For the numerical analysis we select $10^{-2} \le x_p \le 10^{-1}$ where the structure function is flat in x and independent of Q^2. The additional factor in the square bracket originates from the creation of pairs. For large values of x the exponential function is one and this term vanishes. For small x, where the multiplicity is large, the exponential function vanishes and the second term assumes its full strength. Finally, eq. (17) implicitly contains the assumption that the hard gluon and the tube-like (soft-gluon) effects add up incoherently.

The distribution of n-pairs created from the energy stored in the vacuum is, according to eq. (15), a Poisson distribution. This functional form follows from the property that the creation of pairs in each cell is independent of what happens in the other cells. It is also independent of the specific form of $\omega(y)$. The detailed properties of the pairs give information beyond the quark distribution functions.* One consequence is the calculation

*The analogous information for the perturbative part, like the probability $P(N)$ of finding a configuration of N partons in the proton or the joint probability of finding partons with longitudinal fractions $x_1, \cdots, x_N$, cannot be calculated as yet [11].

of the multiplicity

$$n(x) = \sum_{n=1}^{\infty} n P_n = \lambda^2 T \int_0^{L(x)} \omega(y) dy \,. \tag{18}$$

Upon substitution in eq. (17) we obtain

$$q_i^{total}(x, Q^2) = q_i^{pert.}(x, Q^2) + q_i^0(x_p, Q^2)(1 - e^{-n(x)}) \,. \tag{19}$$

The perturbative term from QCD produces the plateau observed at $10^{-2} \leq x_p \lesssim 10^{-1}$. There are several extensions of the perturbative term to smaller values of x. Among them we must select one and add on top of it the non-perturbative contribution of the pairs.

We can give an estimate of the effect. We consider the case $n\frac{B}{E}\pi \ll 1$ and a constant chromoelectric field. In this case the exponent is

$$n(x) = \lambda^2 T \int_0^{L(x)} \omega(y) dy = \lambda^2 T \left\{ \frac{\alpha_s E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{\frac{-n\pi m^2}{|gE|}} \right\} L(x) \,. \tag{20}$$

Studies of particle production give the values [12]

$$\frac{1}{2} g_s E = 0.354 GeV^2 \quad \text{and} \quad \lambda = \sqrt{\pi} \cdot 2.5 \left(\frac{1}{GeV} \right) \,. \tag{21}$$

For these values and $m = m_\pi$ the sum in eq. (20) is close to 0.80. We use the uncertainty principle to estimate the lifetime of a virtual pair as $T \approx \frac{1}{\langle E_{pair} \rangle}$ with $\langle E_{pair} \rangle$ the average energy of a pair. We obtain

$$\begin{aligned} & n(x) \approx 0.005 \cdot \frac{\pi}{x} \qquad \text{for} \qquad \langle E_{pair} \rangle = 1.0 \; GeV \\ \text{and} \quad & n(x) \simeq 0.003 \cdot \frac{\pi}{x} \qquad \text{for} \qquad \langle E_{pair} \rangle = 1.5 \; GeV. \end{aligned} \tag{22}$$

which implies that pair creation begins to become important for $x \sim$ few times 10^{-3}. The estimate is very crude, because it could be modified by the details of the tube, but still encouraging because $n(x)$ begins to grow at a value of x close to the value where the increase of $F_2(x, Q^2)$ is observed. Alternatively, we can assume a functional form for $n(x)$ and calculate the structure function. A possible functional form is $n(x) = f(x) + \frac{c}{x}$ with $f(x)$ a slowly varying function of x and c a constant.

To sum up, the increase observed in $F_2(x, Q^2)$ may originate from the creation of pairs from the vacuum. Consequently the increase of $F_2(x, Q^2)$ from perturbative QCD can be relatively smaller. As a result a new analysis of the data is suggested in terms of

two components: a slow increase from perturbative QCD and a faster increase from the creation of pairs. The limiting value of $F_2^{non-pert.}(x)$ at $x = 0$ is in the present theory finite. Summing the contributions from all the quarks we obtain

$$F_2^{total}(x, Q^2) = F_2^{pert.}(x, Q^2) + F_2(x_p, Q^2)(1 - e^{-n(x)}) \tag{23}$$

with $F_2^{pert.}(x, Q^2)$ the perturbative development of the structure function and $F_2(x_p, Q^2)$ the structure function measured at $10^{-2} \leq x_p \leq 10^{-1}$.

The observed increase in $F_2(x, Q^2)$ is closely related to the increase in the particle multiplicity as x decreases. The particle multiplicity as a function of x has not been reported yet. The data, however, is available and it is interesting to study the correlation between the structure function and the multiplicity expressed in eqs. (19) and (23). The analysis should plot $n(x)\, vs\, x$ and identify a component at $x \leq 10^{-3}$ which originates from the pairs. Plotting $n(x)\, vs\, x$ we expect a faster increase setting in a $x \leq 10^{-3}$.

Ideas describing non-vanishing colour fields in the QCD vacuum have been recognized long time ago and several quantities have already been studied [8], [12]–[18]. The novel aspect of this work is the explanation of the deep inelastic data at small-x in terms of a chromoelectric tube whose length is determined by the scaling variable x and the creation of quark-antiquark pairs by the gluonic field stored in the tube. The creation of pairs is a non-perturbative effect which cannot be produced by the exchange of a finite number of gluons; it comes about as the cumulative effect of infinite many gluons.

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